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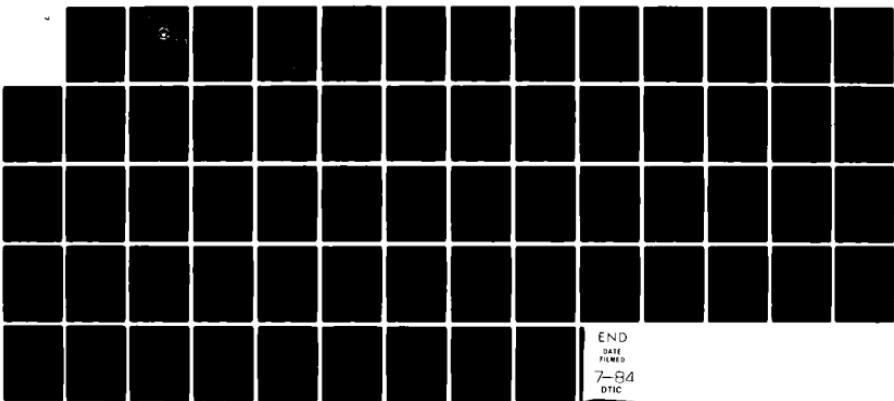
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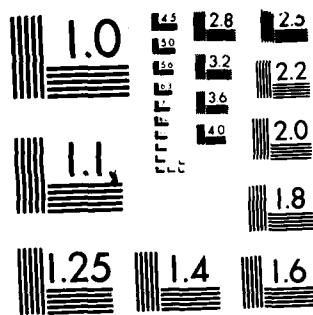
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THESIS

THE EFFECT OF COMPONENT REDUNDANCY
UPON AIRCRAFT COMBAT SURVIVABILITY

by

Angelo J. Kangalos

December 1983

Thesis Advisor: R. E. Ball

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Aircraft combat survivability has developed into a discipline of its own. This thesis presents a brief overview of the two integral parts of survivability, susceptibility and vulnerability. It examines the probability of an aircraft being killed given a single shot fired by the enemy. It further examines the redundancy of critical components and the effect redundancy has on the probability of kill of the aircraft. Several numerical examples are given. The thesis also examines the

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The Effect of Component Redundancy
Upon Aircraft Combat Survivability

by

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Lieutenant Commander, United States Navy
B.S., Youngstown State University, 1970

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN AERONAUTICAL ENGINEERING

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ABSTRACT

Aircraft combat survivability has developed into a discipline of its own. This thesis presents a brief overview of the two integral parts of survivability, susceptibility and vulnerability. It examines the probability of an aircraft being killed given a single shot fired by the enemy. It further examines the redundancy of critical components and the effect redundancy has on the probability of kill of the aircraft. Several numerical examples are given. The thesis also examines the Antiaircraft Artillery Simulation Computer Program, and the modifications required in order to introduce component redundancy into it are developed.

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I.   INTRODUCTION

Since World War I, aircraft combat survivability has become a factor of constantly increasing importance to both pilots and aircraft designers alike. The definition of aircraft combat survivability which will be adopted in this study is "the capability of an aircraft to avoid and/or withstand a man-made, non-nuclear hostile environment" [1].

Given an aircraft flying in a hostile environment, there is a probability that the aircraft will be hit by a damage causing mechanism and a different probability, that the damage causing mechanism will have the catastrophic effect of killing the aircraft. These two probabilities are of paramount importance, and they will be given special attention.

The probability that the aircraft will be hit, P_H, is usually referred to as the susceptibility of the aircraft, while the probability that the aircraft will be killed, given that it has been hit, $P_{K/H}$, is called the vulnerability of the aircraft.

A.   SUSCEPTIBILITY

Assuming that the enemy is alert and possesses weapons, aircraft susceptibility is influenced by the enemy's ability to detect, identify and track the aircraft; launch and

direct threat propagators toward the aircraft; and detonate them at or near the aircraft. If one identifies two phases during the flight of the aircraft, i.e., prelaunch and after launch of the threat propagator, then these abilities can be measured by P_T , the probability that the aircraft has been detected, identified and tracked, and by P_D , the probability that a threat propagator is being launched and detonated at or near the aircraft. Thus, susceptibility P_H is given by

$$P_H = P_T * P_D \qquad (1)$$

B. VULNERABILITY

Aircraft vulnerability is primarily influenced by the aircraft's design. Any survivability features built into the aircraft, such as redundancy, armor plating, and the ability of critical components to operate after taking a hit, will decrease its vulnerability, $P_{K/H}$.

The probability of the aircraft being killed, P_K , is the product of its susceptibility and vulnerability. Thus

$$P_K = P_H * P_{K/H} \qquad (2)$$

P_K is related to the probability of survival of the aircraft, P_s , by

$$P_s = 1 - P_K \qquad (3)$$

This study will examine the computation of the probability of kill of an aircraft and how it is effected by redundancy and the number of hits that the aircraft has taken.

II. PROBABILITY OF KILL GIVEN A SINGLE SHOT

The probability of an aircraft being killed given a
single shot, P_{KSS} , is based on the assumptions that the
aircraft has been detected, and that a threat propagator has
been fired or launched. With these assumptions, the P_{KSS}
becomes solely a function of the fire control system's
ability to direct the threat propagator in the vicinity of
the aircraft and the aircraft's vulnerability. Thus, the
expression for the P_{KSS} in a one-dimensional analysis is

$$P_{KSS} = \int_{-\infty}^{\infty} \rho H(x) * P_{K/H}(x) \, dx \qquad (4)$$

and in a two-dimensional analysis is

$$P_{KSS} = \int_{-\infty}^{\infty} \int_{0} \rho H(x,y) * P_{K/H}(x,y) \, dxdy \qquad (5)$$

where $\rho_H(x)$ is the probability of hit density function at
x , and $P_{K/H}(x)$ is the probability that the aircraft will
be killed given that it has been hit at x .

A. PROBABILITY OF HIT DENSITY FUNCTION

The probability of hit density function is the mathe-
matical expression of the ability of a fire control system

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to direct a threat propagator in the vicinity of the aircraft. In most P_{KSS} assessments, the normal or Gaussian density function is used. If the propagator paths are uniformly distributed around the aircraft, the one-dimensional circular normal is applicable. This function has the form

$$\rho_{H(r)} = \frac{r}{\sigma^2} \exp \left(- \frac{r^2}{2\sigma^2} \right) \qquad 0 \leq r \leq \infty \qquad (6)$$

where r is the propagator miss distance (with respect to $r = 0$), and σ is the standard deviation, which is a measure of the dispersion of the shots about the target. This density function $\rho_{H(r)}$ is used to determine the probability that any shot will have a miss distance that lies within a certain band defined by r_1 and r_2. This probability is given by

$$P_H \Big|_{r_1}^{r_2} = \int_{r_1}^{r_2} \frac{r}{\sigma^2} \exp \left(- \frac{r^2}{2\sigma^2} \right) dr \qquad (7)$$

It is important to note that the probability that any shot lies within the band $r_1 = 0$ and $r_2 = \infty$ is unity.

In the two-dimensional analysis, the bivariate normal distribution in the x-y intercept plane is used for the

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probability of hit density function. This two-dimensional
normal can be given in the form

$$\rho_{H(x,y)} = \frac{1}{2\pi\sigma_x\sigma_y} \exp\left\{-\frac{1}{2}\left[\left(\frac{x-\mu_x}{\sigma_x}\right)^2 + \left(\frac{y-\mu_y}{\sigma_y}\right)^2\right]\right\} \quad (8)$$

where σ_x and σ_y are the standard deviations in the x
and y directions, respectively, and μ_x and μ_y are the
mean values of the shots about the x and y axes,
respectively. The mean value represents a bias or aiming
error and is given by

$$\mu_x = \frac{1}{N}\sum_{i=1}^{N} x_i \quad , \quad \mu_y = \frac{1}{N}\sum_{i=1}^{N} y_i$$

where N is a large number of shots and x_i , y_i are the
x and y coordinates of the ith miss distance.

B. PROBABILITY OF KILL FUNCTION GIVEN A HIT

For contact warheads, the aircraft vulnerability can be
represented by the vulnerable area A_v , where any hit within
the vulnerable area causes an aircraft kill. In the one-
dimensional analysis, the vulnerable area becomes a circle
with a radius given by

$$r_v = \sqrt{\frac{A_v}{\pi}}$$

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and the $P_{K/H}$, or kill function is given by

$$P_{K/H} = 1 \quad \text{for} \quad 0 \leq r \leq r_v$$
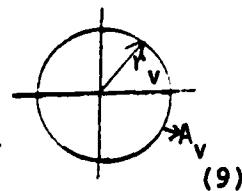
$$= 0 \quad \text{for} \quad r > r_v \tag{9}$$

This type of kill function is referred to as a <u>Cookie Cutter</u> kill function.

Another kill function of particular interest has the form

$$P_{K/H} = \exp \left(- \frac{r^2}{r_v^2} \right) \tag{10}$$

and is referred to as a Carleton kill function. Note, that once again, the area under this $P_{K/H}$ curve is the vulnerable area A_v .

For the two-dimensional analysis, the circular Carleton kill function becomes

$$P_{K/H}(x,y) = \exp \left\{ - \left[\left(\frac{x}{r_v} \right)^2 + \left(\frac{y}{r_v} \right)^2 \right] \right. \tag{11}$$

where r^2 has been replaced by $x^2 + y^2$ in equation (10).

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Substituting equations (6) and (9) into equation (4) and evaluating the integral, the one-dimensional probability of kill given a single shot for the Cookie Cutter kill function becomes

$$P_{KSS} = 1 - \exp\left(-\frac{A_v}{2\pi\sigma^2}\right) \qquad (12)$$

When equations (6) and (10) are substituted into equation (4), the one-dimensional probability of kill given a single shot for the Carleton kill function becomes

$$P_{KSS} = \frac{(\frac{A_v}{2\pi})}{(\frac{A_v}{2\pi}) + 2\sigma^2} \qquad (13)$$

When equations (8) and (11) are substituted into equation (4), the two-dimensional probability of kill given a single shot for the Carleton kill function becomes

$$P_{KSS} = \frac{\frac{A_v}{2\pi}}{\sqrt{\sigma_x^2 + \frac{A_u}{2\pi}} * \sqrt{\sigma_y^2 + \frac{A_v}{2\pi}}} \exp\left\{-\frac{1}{2}\left[\frac{\mu_x^2}{\sigma_x^2 + \frac{A_u}{2\pi}} + \frac{\mu_y^2}{\sigma_y^2 + \frac{A_v}{2\pi}}\right]\right\} \qquad (14)$$

If one assumes $A_v << \sigma_x^2, \sigma_y^2$ then

$$P_{KSS} = \frac{A_v}{2\pi\sigma_x\sigma_y} \exp\left[-\frac{1}{2}\left(\frac{\mu_x^2}{\sigma_x^2} + \frac{\mu_y^2}{\sigma_y^2}\right)\right] \qquad (15)$$

Thus, according to equation (15), P_{KSS} is a linear function of the aircraft's vulnerable area.

Detailed evaluations of equations (12), (13) and (14) are included in Appendix A.

III. REDUNDANCY AND THE P_{KSS}

As it was previously defined, vulnerability is the inability of an aircraft to withstand one or more hits by damage causing mechanisms. The higher the vulnerability of an aircraft, the worse its chances of survival are. Thus, one might say that vulnerability is a disadvantage in an aircraft. In order to reduce the vulnerability and overcome this disadvantage, designers are utilizing six vulnerability reduction concepts [1], one of which is component redundancy.

An aircraft is comprised of a multitude of components, and each one of them contributes to the overall aircraft vulnerability. Some components contribute much more than the others, and for that reason they are called critical components. Thus, a component is categorized as critical, if a hit by a damage causing mechanism on the component leads to an aircraft kill. If an aircraft is comprised of N non-redundant critical components, then the aircraft will survive if and only if each of the N critical components survives. Thus, the aircraft's probability of survival after the first hit, $P_s^{(1)}$, will be

$$P_s^{(1)} = P_{s_1}^{(1)} * P_{s_2}^{(1)} * P_{s_3}^{(1)} * \ldots * P_{s_N}^{(1)} \qquad (16)$$

where $P_{s_1}^{(1)}$, $P_{s_2}^{(1)}$, $P_{s_3}^{(1)}$...$P_{s_N}^{(1)}$ are the probabilities of survival of critical component 1,2,3,...N , after the first shot. If one takes into consideration equation (3) the probability of aircraft survival could be written as

$$P_s^{(1)} = \prod_{i=1}^{N} P_{s_i}^{(1)} = \prod_{i=1}^{N} (1 - P_{k_i}^{(1)}) \qquad (17)$$

where $P_{s_i}^{(1)}$ is the probability of survival of the ith component after the first shot, and $P_{k_i}^{(1)}$ is the probability of kill of the ith component after the first shot.

If there are sets of redundant critical components, the validity of equation (16) breaks down, that is, a hit on a redundant critical component does not constitute an aircraft kill, since another component could sustain the aircraft flight. This can be best illustrated by the following example aircraft, which is comprised of four critical components, the pilot, the fuel tank and the redundant set of engines one and two. (The aircraft is assumed to be capable of flying on only one engine.) The probability that the aircraft will survive the first shot is

$$P_s^{(1)} = P_{s_p}^{(1)} * P_{s_F}^{(1)} * P_{s_{EE}}^{(1)} \qquad (18)$$

where $P_{s_p}^{(1)}$, $P_{s_F}^{(1)}$ and $P_{s_{EE}}^{(1)}$ are the probabilities of survival of the pilot, the fuel tank, and either one or both engines respectively, since both engines must be killed

simultaneously in order to kill the aircraft. The pro-
bability that either engine one or engine two or both
engines survive after the first shot is given by

$$P_{s_{EE}}^{(1)} = 1 - P_{k_{E1}}^{(1)} * P_{k_{E2}}^{(1)} \qquad (19)$$

where $P_{k_{E1}}$ and $P_{k_{E2}}$ are the probability of kill of
engines one and two, respectively, and the product of
$P_{k_{E1}}$ and $P_{k_{E2}}$ is the probability that both engines will
be killed by the first shot. This last concept is an
interesting one and will now be investigated further.

In the following investigation, the one-dimensional
expression for the Cookie Cutter and Carleton kill functions
will be utilized along with the one-dimensional provability
of hit density function. First, using the Cookie Cutter
approach, figure 1 represents the situation for the vulner-
ability of two separate redundant critical components.
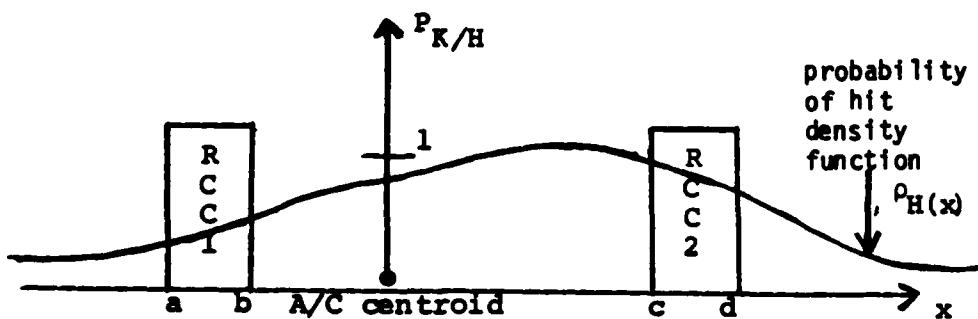
Figure 1.   Vulnerability of Two Separate Redundant Criti-
cal Components, Cookie Cutter Approach

In figure 1, the x-axis represents the aircraft length
(vice area, since one-dimensional analysis is used), a-b
is the vulnerable length of redundant component one, and
c-d is the vulnerable length of redundant component two.
In this approach, the kills of redundant components one and
two are mutually exclusive events; if component one is
killed on the hit, then component two is still alive and
vice versa, since only one vulnerable length could be hit
at one time.

With the Carleton kill function shown in figure 2
everything changes. As seen from figure 2, the kills of
redundant critical components one and two are not mutually
exclusive events.
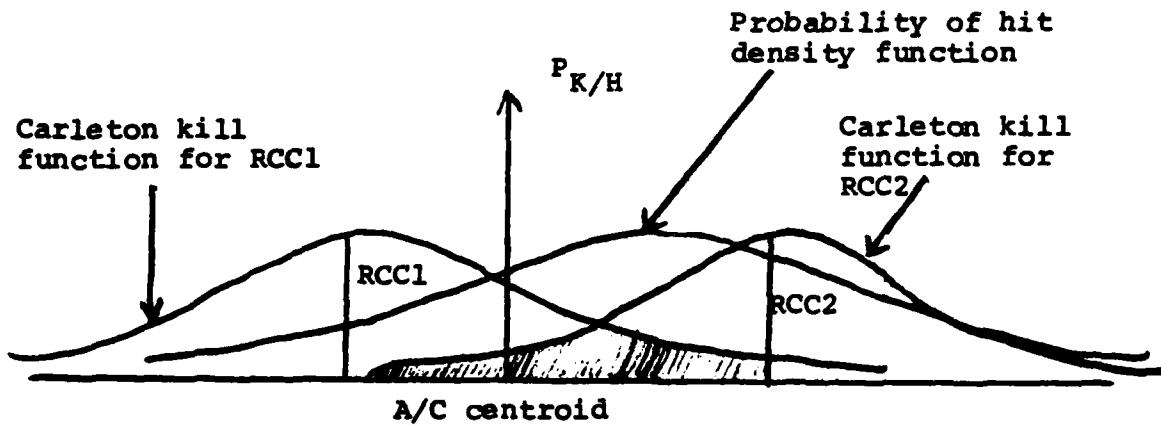
Figure 2. Vulnerability of Two Separate Redundant Critical
Components, Carleton Approach

A hit anywhere from $-\infty$ to $+\infty$ will have a non-zero probability of killing both redundant critical components. In essence, the Carleton kill function creates an overlap of the components. Thus, one hit can possibly kill the aircraft because both redundant components can be killed by one hit.

Returning back to the example aircraft, equation (18) becomes

$$P_s^{(1)} = (1 - P_{k_p}^{(1)}) * (1 - P_{k_F}^{(1)}) * (1 - P_{k_{E1}}^{(1)} * P_{k_{E2}}^{(1)}) \qquad (20)$$

This equation is valid for an aircraft with one set of two redundant critical components where only a kill of both of them will result in an aircraft kill. There are some other combinations of redundant critical components that one might encounter in a kill tree analysis. They are

(1) A set of three-redundant critical components and only a kill of all three will result in an aircraft kill. For this particular combination, the aircraft kill logical expression due to just the redundant set will be

$$\text{A/C KILL}_{RC} = (RC1.AND.RC2.AND.RC3)$$

while the probability of any one, any two or all three components surviving the first shot, will be given by

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$$P_{s_{RC}}^{(1)} = 1 - P_{k_{RC1}}^{(1)} * P_{k_{RC2}}^{(1)} * P_{k_{RC3}}^{(1)} \qquad (21)$$

where $P_{k_{RC}}^{(1)}$, $P_{k_{RC2}}$, $P_{k_{RC3}}$ are the probabilities of kill for redundant critical components one, two and three, respectively.

(2) A set of three redundant critical components and a kill of any two out of three will result in an aircraft kill. The logical expression for this combination is

A/C KILL$_{RC}$ = (RC1.AND.RC2).OR. (RC2.AND.RC3).OR. (RC1.AND.RC3)

while the probability of any two or all three redundant critical components, surviving the first shot will be

$$P_{s_{RC}}^{(1)} = 1 - P_{k_{RC1}}^{(1)} * P_{k_{RC2}}^{(1)} - P_{k_{RC2}}^{(1)} * P_{k_{RC3}}^{(1)}$$
$$\qquad (22)$$
$$- P_{k_{RC1}}^{(1)} * P_{k_{RC3}}^{(1)} + P_{k_{RC1}}^{(1)} * P_{k_{RC2}}^{(1)} * P_{k_{RC1}}^{(1)}$$

(3) A set of four redundant critical components and a kill of any two out of four will result in an aircraft kill. The logical expression for this combination will be

A/C KILL$_{RC}$ = (RC1.AND.RC2).OR.(RC1.AND.RC3).OR.(RC1.AND.RC4)

.OR.(RC2.AND.RC3).OR.(RC2.AND.RC4).OR.(RC3.AND.RC4)

while the probability of any two, three or all four critical components surviving the first shot will be.

$$P_{s_{RC}}^{(1)} = 1 - P_{k_{RC1}}^{(1)} * P_{k_{RC2}}^{(1)} - P_{k_{RC1}}^{(1)} * P_{k_{RC3}}^{(1)}$$

$$- P_{k_{RC1}}^{(1)} * P_{k_{RC4}}^{(1)} - P_{k_{RC2}}^{(1)} * P_{k_{RC3}}^{(1)}$$

$$- P_{k_{RC2}}^{(1)} * P_{k_{RC4}}^{(1)} - P_{k_{RC3}}^{(1)} * P_{k_{RC4}}^{(1)}$$

$$+ P_{k_{RC1}}^{(1)} * P_{k_{RC2}}^{(1)} * P_{k_{RC3}}^{(1)} + P_{k_{RC1}}^{(1)} * P_{k_{RC2}}^{(1)} * P_{k_{RC4}}^{(1)}$$

$$+ P_{k_{RC1}}^{(1)} * P_{k_{RC3}}^{(1)} * P_{k_{RC4}}^{(1)} + P_{k_{RC2}}^{(1)} * P_{k_{RC3}}^{(1)} * P_{k_{RC4}}^{(1)}$$

$$+ P_{k_{RC1}}^{(1)} * P_{k_{RC2}}^{(1)} * P_{k_{RC3}}^{(1)} * P_{k_{RC4}}^{(1)} \tag{23}$$

(4)   A set of four redundant critical components and a kill of any three out of four will result in an aircraft kill. The logical expression for this combination is

A/C KILL$_{RC}$ = (RC1.AND.RC2.AND.RC3).OR.(RC1.AND.RC2.AND.RC4).
.OR.(RC1.AND.RC3.AND.RC4)
.OR.(RC2.AND.RC3.AND.RC4)

while the probability of any one, two, three or all four redundant components surviving the first shot will be

$$P_{s_{RC}}^{(1)} = 1 - P_{k_{RC1}}^{(1)} * P_{k_{RC2}}^{(1)} * P_{k_{RC3}}^{(1)}$$

$$- P_{k_{RC1}}^{(1)} * P_{k_{RC2}}^{(1)} * P_{k_{RC4}}^{(1)}$$

$$- P_{k_{RC1}}^{(1)} * P_{k_{RC3}}^{(1)} * P_{k_{RC4}}^{(1)}$$

$$- P_{k_{RC2}}^{(1)} * P_{k_{RC3}}^{(1)} * P_{k_{RC4}}^{(1)}$$

$$+ P_{k_{RC1}}^{(1)} * P_{k_{RC2}}^{(1)} * P_{k_{RC3}}^{(1)} * P_{k_{RC4}}^{(1)} \qquad (24)$$

(5)   A set of four redundant critical components and a kill of all four only will result in an aircraft kill. The logical expression for this combination will

$$\text{A/C KILL}_{RC} = RC1.AND.RC2.AND.RC3.AND.RC4$$

while the probability that any one, two, three or all four redundant components will survive the first shot will

$$P_{s_{RC}}^{(1)} = 1 - P_{k_{RC1}}^{(1)} * P_{k_{RC2}}^{(1)} * P_{k_{RC3}}^{(1)} * P_{k_{RC4}}^{(1)} \quad (25)$$

IV. $P_{K/H}$ AND REDUNDANCY

The analysis done in the previous chapter, though
necessary, was limited to the probability that an aircraft
is killed due to a single shot. A further and more
realistic analysis stems from the fact that in any given
combat scenario, an aircraft is likely to take multiple
hits. Thus, the total probability of kill of an aircraft
that has taken multiple hits will now be examined.

If one assumes that an aircraft consists of N critical
components and that it has taken n hits is given by

$$P_{s_i}^{-(n)} = P_{s_i}^{(1)} * P_{s_i}^{(2)} * P_{s_i}^{(3)} *...P_{s_i}^{(n)} = \prod_{J=1}^{n} P_{s_i}^{(j)} \qquad (26)$$

where $P_{s_i}^{(1)}$, $P_{s_i}^{(2)}$, $P_{s_i}^{(3)}...P_{s_i}^{(n)}$ are the probabil-
ities that the ith critical component will survive the
first, second, third...nth hit, respectively. Using a
parallel which may be inferred from equation (17),

$$P_{s_i}^{-(n)} = \prod_{J=1}^{n} (1 - P_{k/h_i}^{j}) \qquad (27)$$

where P_{k/h_i} is the probability that the ith component will
be killed by the jth hit. In a similar manner, the total
probability of aircraft survival after n hits can be
derived to yield

$$\overline{P}_s{}^{(n)} = \prod_{j=1}^{n} (1 - P_{K/H}{}^j) \tag{28}$$

where $P_{K/H}{}^j$ is the probability that the aircraft will be killed by the jth hit.

In the case of an aircraft consisting of only non-redundant critical components, the $P_{K/H}$ does not change with each hit because the vulnerable area is constant for each hit. However, in the case of redundant critical components, $P_{K/H}$ changes considerably due to the increasing probability the redundant critical components are killed and thus, it will be studied separately.

The simplest approach to the $P_{K/H}$ with redundancy study is through the use of a kill tree diagram. For that, assume an A/C consisting of three non-redundant critical components CC1, CC2 and CC3, with individual probabilities of kill P_{K/H_1}, P_{K/H_2}, P_{K/H_3}, respectively, and or set of two redundant critical components RCC1 and RCC2 with individual probabilities of kill $P_{K/H_{R_1}}$ and $P_{K/H_{R_2}}$, respectively. Thus, the kill diagram is depicted on figure 3 for the first hit on the aircraft and the $P_{K/H}{}^{-(1)}$ for the Cookie Cutter approach becomes

$$P_K{}^{-(1)} = P_{K/H_1} + P_{K/H_2} + P_{K/H_3} \tag{29}$$

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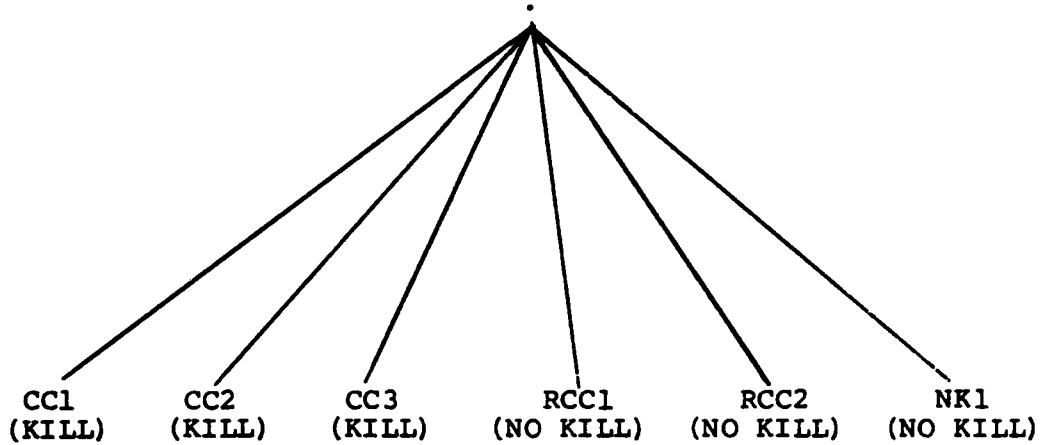
Figure 3.   Aircraft Kill Tree Diagram, First Hit

In order for the A/C to be killed by the second hit, one must assume that it survived the first hit, and therefore only the RCC1, RCC2 and NK1 branches will be affected by the second hit.   Thus, the kill tree for the second hit alone is depicted on figures 4a, 4b and 4c.
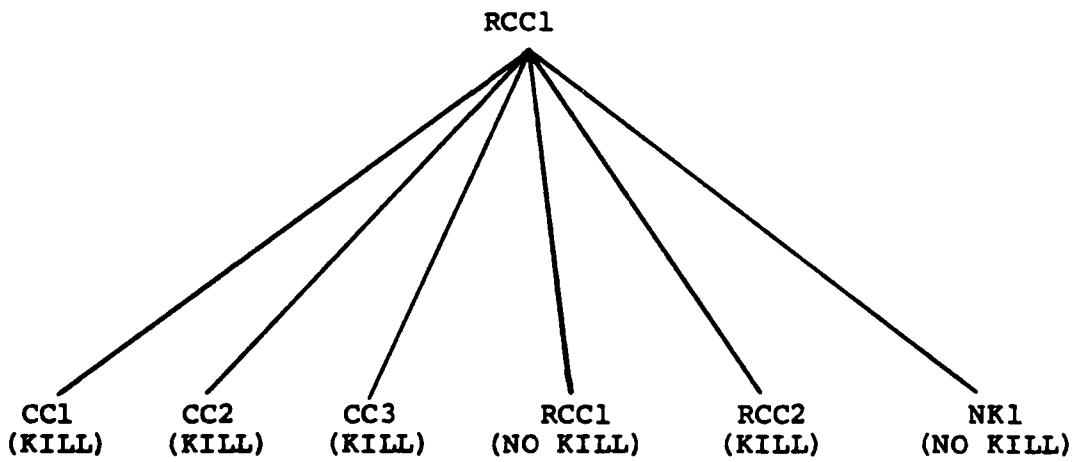
Figure 4a.   Aircraft Kill Tree Diagram, Second Hit Alone
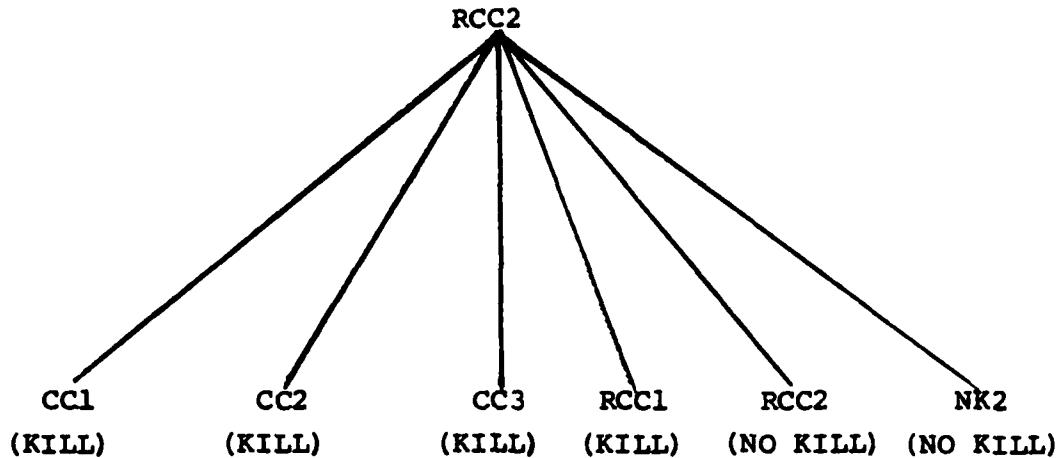
Figure 4b.   Aircraft Kill Tree Diagram, Second Hit Alone
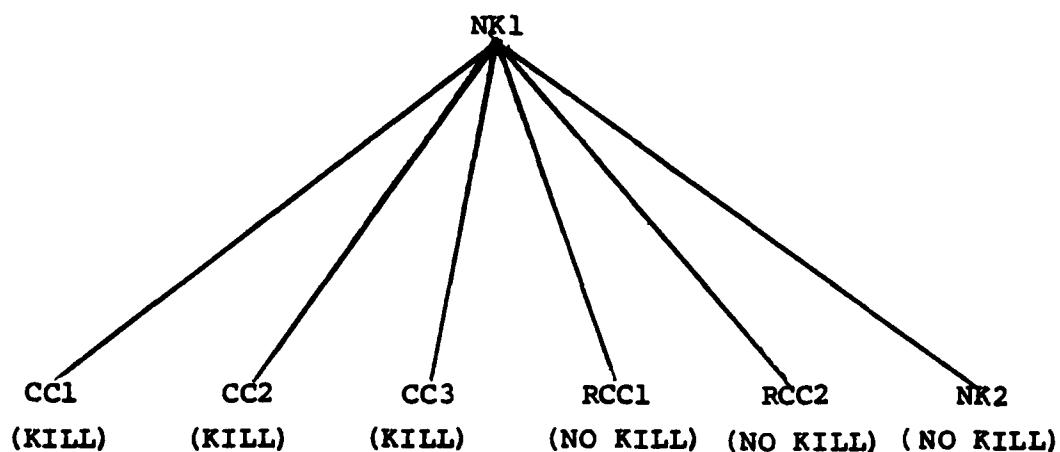
Figure 4c.   Aircraft Kill Tree Diagram, Second Hit Alone

Thus, the probability of the A/C being killed by the second
hit alone becomes

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$$P_K^{(2)} = P_{k/H_{R1}}^{(1)} * \left(P_{k/H_1}^{(2)} + P_{k/H_2}^{(2)} + P_{k/H_3}^{(2)} + P_{k/H_{R2}}^{(2)} \right)$$

$$+ P_{k/H_{R2}}^{(1)} * \left(P_{k/H_1}^{(2)} + P_{k/H_2}^{(2)} + P_{k/H_3}^{(2)} + P_{k/H_{R1}}^{(2)} \right)$$

$$+ P_s^{(1)} * \left(P_{k/H_{R1}}^{(2)} * P_{k/H_{R2}}^{(2)} \right) \tag{30}$$

where $P_s^{(1)}$ is the probability that all of the critical components survived the first hit (the NK1 branch). Thus, the total probability of kill for the two hits $P_{K/H}^{-(2)}$ becomes

$$P_K^{-(2)} = P_K^{-(1)} + P_K^{(2)} \tag{31}$$

One could go on to the third and subsequent hits in a similar manner, and the $P_{K/H}$ will be increasing as more and more terms are added for the subsequent hits.

Another method of computing the total probability of kill is the transition matrix method [2]. Using that method on the following example aircraft, one can clearly see the effect of redundancy on the $P_{K/H}$.

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A/C #1 (NO REDUNDANCY)

CRITICAL COMPONENT	A_{p_i} (ft^2)	P_{k/h_i}	A_{v_i} (ft^2)
Pilot	5	1.0	5
Engine	30	.8	24
Fuel Tank	40	.6	24
Controls	25	.6	15

Total aircraft area present to the threat propagator is assumed to be $A_p = 500$ ft^2 , and A_{p_i} is the area presented by the ith component, P_{k/h_i} is the probability of ith component given a hit on the ith component, and A_{v_i} is the vulnerable area of the ith component. The logical kill expression for this aircraft will be

A/C KILL = PILOT.OR.ENGINE.OR.FUEL.OR.CONTROLS

A/C #2 (25% REDUNDANT) $A_p = 500$ ft^2

CRITICAL COMPONENT	A_{p_i} (ft^2)	P_{k/h_i}	A_{v_i} (ft^2)
Pilot	5	1.0	5
Engine	30	.8	24
Fuel Tank	40	.6	24
Controls I	12.5	.6	7.5
Controls II	12.5	.6	7.5

The logical expression for this aircraft is

A/C KILL = PILOT.OR.ENGINE.OR.FUEL.OR.(CONTROLI.AND.CONTROLII)

A/C #3 (46% REDUNDANT) A_p = 530 ft^2

CRITICAL COMPONENT	A_{p_i} (ft^2)	P_k/h_i	A_{v_i} (ft^2)
Pilot	5	1.0	5
Engine I	30	.8	24
Engine II	30	.8	24
Fuel Tank	40	.6	24
Controls	25	.6	15

The logical kill expression for this aircraft is

A/C KILL = PILOT.OR.FUEL.OR.CONTROLS.OR.(ENGINEI.AND.
ENGINEII)

A/C #4 (77% REDUNDANT) A_p = 530 ft^2

CRITICAL COMPONENT	A_{p_i} (ft^2)	P_k/h_i	A_{v_i} (ft^2)
Pilot	5	1.0	5
Engine I	30	.8	24
Engine II	30	.8	24
Fuel Tank I	20	.5	10
Fuel Tank II	20	.5	10
Controls	25	.6	15

The logical expression for A/C #4 is

A/C KILL = PILOT.OR.CONTROLS.OR.(ENGINEI.AND.ENGINEII).
OR.(FUELI.AND.FUELII)

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TABLE 1

CUMULATIVE PROBABILITIES OF KILL FOR EXAMPLE AIRCRAFT

		A/C #1	A/C #2	A/C #3	A/C #4
$P_k^{-(1)}$	=	.1360	.1060	.0830	.0377
$P_k^{-(2)}$	=	.2535	.2012	.1632	.0788
$P_k^{-(3)}$	=	.3550	.2866	.2397	.1220
$P_k^{-(4)}$	=	.4227	.3633	.3117	.1662
$P_k^{-(5)}$	=	.5185	.4320	.3788	.2107
$P_k^{-(6)}$	=	.5840	.4934	.4411	.2548
$P_k^{-(7)}$	=	.6406	.5485	.4984	.2981
$P_k^{-(8)}$	=	.6895	.5978	.5508	.3403
$P_k^{-(9)}$	=	.7317	.6418	.5986	.3809
$P_k^{-(10)}$	=	.7682	.6811	.6419	.4199

Figure 5 compares the $\bar{P}_k$ for the nonredundant aircraft #1 with redundant aircraft #2, 3 and 4.
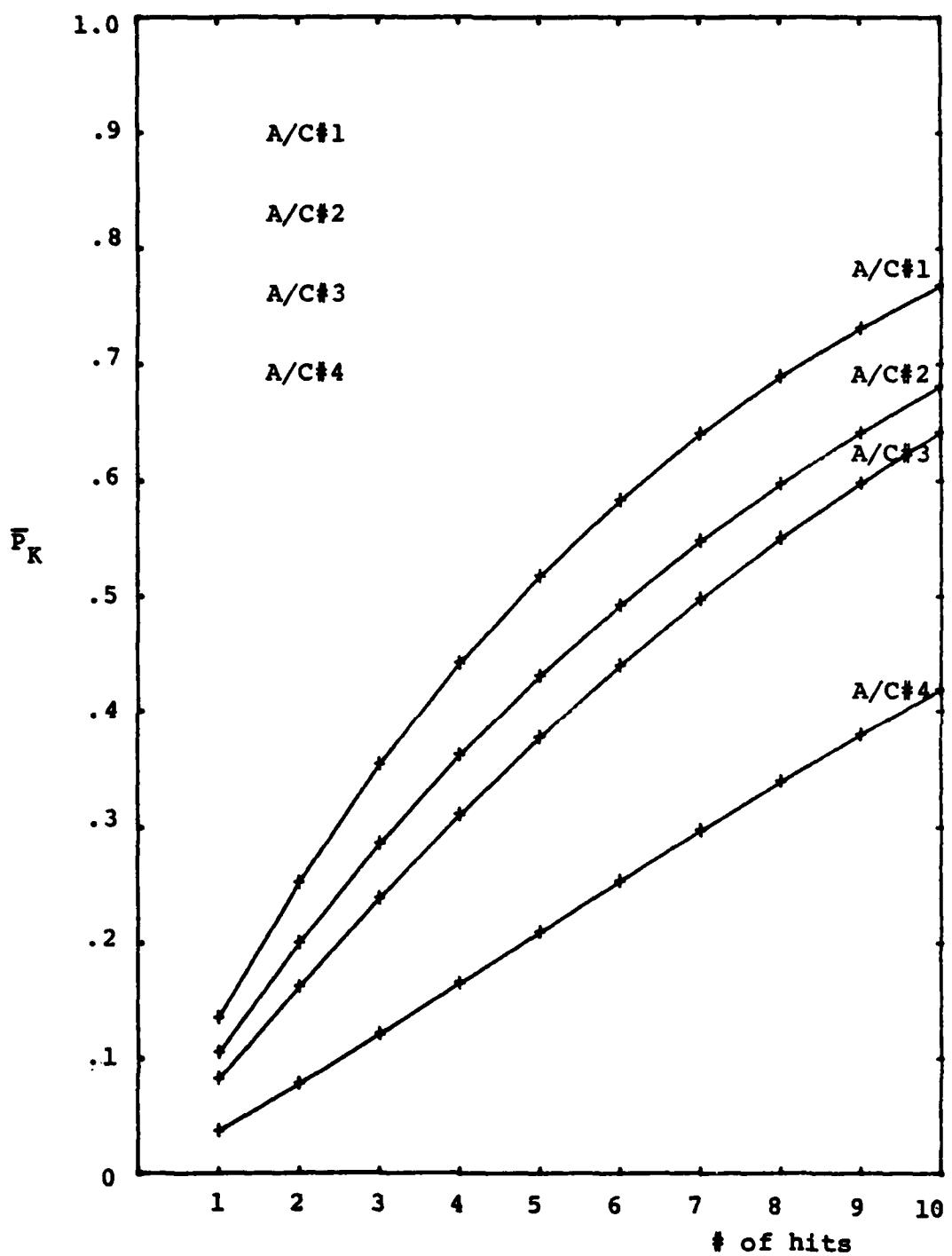
FIGURE 5.   CUMULATIVE PROBABILITY OF KILL VERSUS NUMBER OF HITS

Note that the values of A_{p_i} and P_{k/h_i} are assumed, while the values of A_{v_i} are computed through the equation

$$A_{v_i} = A_{p_i} * P_{k/h_i} \qquad (32)$$

Through the use of the transition matrix method (see Appendix B), the values of $\bar{P}_k^{(n)}$ for the example aircraft have been found and are presented in Table 1.

As can be very easily seen from Table 1, the $\bar{P}_k$ for the same shot decreases, as the redundancy of the example aircraft increases.

V. AFATL POOL COMPUTER PROGRAM AND THE $P_{K/H}$

As shown by the previous chapters, the determination of
the survivability of an aircraft can be a very complex and
time consuming task. Thus, the aid of the computer has been
employed, and several programs have been developed by both
the military and the civilian industry. One that was
selected for use by the Joint Technical Coordinating Group
for Aircraft Survivability (JTCG/AS) Survivability Assessment
Subgroup, is the Antiaircraft Artillery Simulation Computer
Program, AFATL Program POO1. This program is currently in
use at the Naval Postgraduate School and is the one that
this study will be concerned with.

Many modifications have been made to the POO1 that was
originally developed by the Weapons Systems Analysis
Division of the Air Force Armament Laboratory at Eglin Air
Force Base, Florida. The latest version of it used by Naval
Weapons Center, China Lake, employs an aircraft consisting
of twenty (20) critical components and computes the single
shot probability of kill of a target (after consideration
is given to various errors in predicting an aircraft/
projectile intercept point). Computation of the P_{KSS} is
performed throughout the entire flight path and after the
accumulation of all shots, the total probability of target

attrition is presented. In the computation of the cumulative $\bar{P}_K$, redundancy of the critical components has not been taken into consideration. Thi.. is a potential source of error in the final results, since the majority of the present aircraft have some degree of redundancy.

In the latest version of POO1, the computation of the single shot probability of kill is performed in the "FIRE" section of the program. The equation used to compute the probability of killing, at least one critical component by a single round, P_K , is presented on line 717 of the program listing given in Appendix C, and it states

$$\widehat{PK} = \bar{P}K + PKS(IJ) - \bar{P}_K*PKS(IJ) = \bar{P}K + PKS(IJ)*(1 - \bar{P}K)$$

where the initial value of P_K is set to zero and $P_{KS(IJ)}$ is the probability of killing component IJ by a single round. This equation does not account for redundancy because component IJ might be redundant in the aircraft and killing it will not constitute an aircraft kill, as it was discussed in Chapter 3. Thus, in order for the program to account for possible redundancy of components, the following changes and additions to POO1 are recommended.

Define:

IJ1 = the total number of non-redundant components

IJ22 = the total number of sets of <u>two</u> redundant components
of which <u>two</u> must be killed for a subsystem kill

IJ23 = the total number of sets of <u>three</u> redundant components
of which <u>two</u> must be killed for a subsystem kill

IJ33 = the total number of sets of <u>three</u> redundant components
of which <u>three</u> must be killed for a subsystem kill

IJ24 = the total number of sets of <u>four</u> redundant components
of which <u>two</u> must be killed for a subsystem kill

IJ34 = the total number of sets of <u>four</u> redundant components
of which <u>three</u> must be killed for a subsystem kill

IJ44 = the total number of sets of <u>four</u> redundant components
of which <u>four</u> must be killed for a subsystem kill

Thus, $IJ = IJ1 + 2*(IJ22) + 3*(IJ23 + IJ33) + 4*(IJ24 + IJ34 + IJ44) \leq 20$

Define:

MN(I)   = A column matrix containing the numbers of the non-
           redundant components (I=1,IJ1)

M22(I,J) = A (IJ22x2) matrix containing the IJ22 pairs of
           redundant components (I=1,IJ22, J=1,2)

M23(I,J) = A (IJ23x3) matrix containing the IJ23 sets of
           three redundant components (I=1,IJ23, J=1,3)

M33(I,J) = A (IJ33x3) matrix containing the IJ33 sets of
           three redundant components (I=1,IJ33, J=1,3)

M24(I,J) = A (IJ24x4) matrix containing the IJ24 sets of
           four redundant components (I=1,IJ24, J=1,4)

M34(I,J) = A (IJ34x4) matrix containing the IJ34 sets of
           four redundant components (I=1,IJ34, J=1,4)

M44(I,J) = A (IJ44x4) matrix containing the IJ44 sets of
           four redundant components (I=1,IJ44, J=1,4)

The user must create a file containing the number of the
non-redundant components, then redundant components them-
selves, the number of the IJ22,IJ23,IJ33,IJ24,IJ34,IJ44 sets,
and the numbers of the components that belong to each set as
shown below:

```
IJ1                                                      etc. (max # of
MN(1)   MN(IJ1)              M22(IJ22,1)                  characters = 38)
MN(2)   IJ22                 M22(IJ22,2)
        M22(1,1)             IJ23
        M22(1,2)             M23(1,1)
        M22(2,1)             M23(1,2)
        M22(2,2)             M23(1,3)

|_|_|_•_•_•_|_|_|_|_|_|_|_•_|_•_|_|_•_|_|_|_|_|_|_|_|_|_|
    \__IJ1__/   _____2*IJ22_____/  \____3*IJ23____/
```
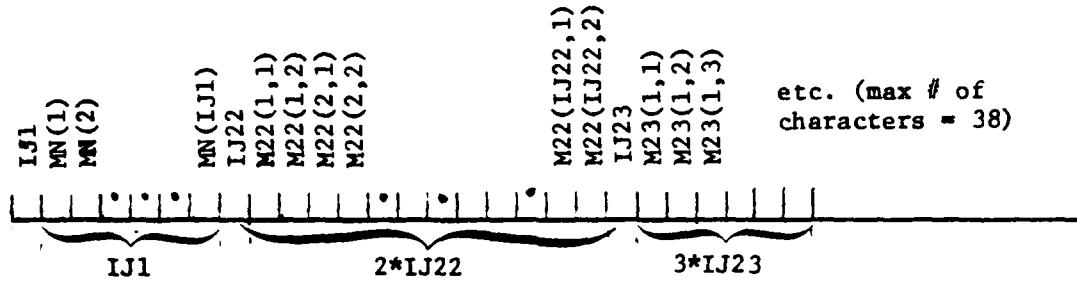
Once the file is constructed, the MN,M22,M23,M33,M24,M34,M44
matrices can be read, with the following routine:

```
      READ(2,*)(TEMP(I),I=1,38)

      I=1

      IJ1=TEMP(I)

      IF IJ1=0     GO TO 6

      DO 5 K=1,IJ1

          MN(K)=TEMP(I+K)

5     CONT

6     I=1+IJ1+1

      IJ22=TEMP(I)

      IF IJ22=0    GO TO 10
```

```
            DO 15 K=1, IJ22

                    M22(K,1)=TEMP(I+K)

                    M22(K,2)=TEMP(I+K+1)

                    I=I+2

15    CONT

10    I=2+IJ1+2*IJ22+1

      IJ23+TEMP(I)

      IF IJ23=0     GO TO 20

      DO 25 K=1,IJ23

                    M23(K,1)=TEMP(I+K)

                    M23(K,2)+TEMP(I+K+1)

                    M23(K,3)=TEMP(I+K+2)

                    I=I+3

25    CONT

20    I=3+IJ1+2*IJ22+3*IJ23+1

      IJ33=TEMP(I)

      IF IJ33 = 0    GO TO 30

      DO 35 K=1,IJ33

                    M33(K,1)=TEMP(I+K)

                    M33(K,2)=TEMP(I+K+1)

                    M33(K,3)=TEMP(I+K+2)

                    I=I+3

35    CONT
```

40

```
30   I=4+IJ1+2*IJ22+3*IJ23+3*IJ33+1

     IJ24=TEMP(I)

     IF IJ24=0    GO TO 40

     DO 45 K=1,IJ24

          M24(K,1)=TEMP(I+K)

          M24(K,2)=TEMP(I+K+1)

          M24(K,3)=TEMP(I+K+2)

          M24(K,4)=TEMP(I+K+3)

          I=I+4

45   CONT

40   I=5+IJ1+2*IJ22+3*IJ23+3*IJ33+4*IJ24+1

     IJ34=TEMP(I)

     IF IJ34=0    GO TO 50

     DO 55 K=1,IJ34

          M34(K,1)=TEMP(I+K)

          M34(K,2)=TEMP(I+K+1)

          M34(K,3)=TEMP(I+K+2)

          M34(K,4)=TEMP(I+K+3)

          I=I+4

55   CONT

50   I=6+IJ1+2*IJ22+3*IJ23+3*IJ33+4*IJ24+4*IJ34+1

     IJ44=TEMP(I)

     IF IJ44=0    GO TO 60
```

```
      DO 65 K=1,IJ44

          M44(K,1)=TEMP(I+K)

          M44(K,2)=TEMP(I+K+1)

          M44(K,3)=TEMP(I+K+2)

          M44(K,4)=TEMP(I+K+3)

          I=I+4

60    STOP

      END
```

Once the matrices have been constructed, the following subroutine could be used to compute the cumulative probability of aircraft survival. The cumulative probability of kill of each of the components (CPKS(IJ)) is computed at line 722. This probability is computed for all of the components in the DO to 460 loop. Immediately after the 460 loop (before line 72) a call to the following subroutine should be made. The parameter computed by this subroutine is the probability the aircraft has survived the encounter up to the current time, PS , the cumulative probability of survival. Note that the PK and PS computed at lines 717, 727, and 732 are no longer valid when redundant components are introduced.

```fortran
      PS=1.

      IF IJ1=0   GO TO 10

      DO 5  K=1,IJ1

          L=MN(K)

          PS=PS*(1.-CPKS(L))

      CPKS is the cumulative probability of survival of the
      component

5     CONTINUE

10    IF IJ22=0   GO TO 20

      DO 15 K=1,IJ22

          L=M22(K,1)

          M=M22(K,2)

          PS=PS*(1.-CPKS(L)*CPKS(M))

15    CONTINUE

20    IF IJ23=0   GO TO 30

      DO 25 K=1,IJ23

          L=M23(K,1)

          M=M23(K,2)

          N=M23(K,3)

          PS=PS*(1.-CPKS(L)*CPKS(M)-CPKS(L)*CPKS(N)
              -CPKS(M)*CPKS(N)+CPKS(L)*CPKS(M)*CPKS(N))

25    CONTINUE
```

```fortran
30   IF IJ33=0  GO TO 40

     DO 35 K=1,IJ33

          L=M33(K,1)

          M=M33(K,2)

          N=M33(K,3)

          PS=PS*(1.-CPKS(L)*CPKS(M)*CPKS(N))

35   CONTINUE

40   IF IJ24=0  GO TO 50

     DO 45 K=1,IJ24

          L=M24(K,1)

          M=M24(K,2)

          N=M24(K,3)

          I=M24(K,4)

          PS=PS*(1.-CPKS(L)*CPKS(M)-CPKS(L)*CPKS(N)-CPKS(L)
     *         *CPKS(I)-CPKS(M)*CPKS(N)-CPKS(M)*CPKS(I)
     *         -CPKS(N)*CPKS(I)+CPKS(L)*CPKS(M)*CPKS(N)
     *         +CPKS(L)*CPKS(M)*CPKS(I)+CPKS(L)*CPKS(N)
     *         *CPKS(I)+CPKS(M)*CPKS(N)*CPKS(I)+CPKS(L)*CPKS(M)
     *         *CPKS(N)*CPKS(I))

45   CONTINUE

50   IF IJ34=0  GO TO 60

     DO 55 K=1,IJ34

          L=M34(K,1)

          M=M34(K,2)

          N=M34(K,3)

          I=M34(K,4)
```

```
              PS=PS*(1.-CPKS(L)*CPKS(M)*CPKS(N)-CPKS(L)*CPKS(M)

                 *CPKS(I)-CPKS(L)*CPKS(N)*CPKS(I)-CPKS(M)

                 *CPKS(N)*CPKS(I)+CPKS(L)*CPKS(M)*CPKS(N)

                 *CPKS(I))

    55    CONTINUE

    60    IF IJ44=0   GO TO 70

          DO 65 K=1,IJ44

              L=M44(K,1)

              M=M44(K,2)

              N=M44(K,3)

              I=M44(K,4)

              PS=PS*(1.-CPKS(L)*CPKS(M)*CPKS(N)*CPKS(I))

    65    CONTINUE

    70    WRITE (6,100) PS

    100   FORMAT ('1,   'THE CUMULATIVE PROBABILITY OF SURVIVAL IS,

          F9.7)

          RETURN

          END
```

VI. CONCLUSIONS

The probability of kill of an aircraft given a single shot is a linear function of the aircraft's vulnerable area. Redundancy of critical components greatly enhances the probability of survival and in the Cookie Cutter approach the kills of redundant critical components are mutually exclusive events, where as in the Carleton approach they are not. For an aircraft, hit by multiple shots the cumulative probability of kill for the same shot decreases as the redundancy built into the aircraft increases. The modifications proposed in Chapter V should provide a significant improvement in the overall efficiency of the Antiaircraft Artillery Simulation Computer Program, since it will provide for aircraft redundancy.

APPENDIX A

EVALUATION OF EQUATIONS

(1) Evaluation of equation (12).

$$P_{KSS} = \int_0^{r_v} (1) * \frac{r}{\sigma} \exp\left(-\frac{r^2}{2\sigma^2}\right) dr \quad (1)$$

Using integration by parts, let $-\frac{r^2}{2\sigma^2} = u$.

Differentiating results in $-\frac{2r}{2\sigma^2} dr = du$

Therefore

$$P_{KSS} = \int - \exp(u) \ du = - \exp\left(-\frac{r^2}{2\sigma^2}\right)\Bigg|_0^{r_v} =$$

$$- \exp\left(-\frac{r_v^2}{2\sigma^2}\right) - (-1) = 1 - \exp \ -\frac{r_v^2}{2\sigma^2}$$

Since $A_v = \Pi r_v^2 \rightarrow P_{KSS} = 1 - \exp\left(-\frac{A_v}{2\Pi\sigma^2}\right)$

(2) Evaluation of equation (13).

$$P_{KSS} = \int_0^\infty \frac{r}{\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2}\right) * \exp\left(-\frac{r^2}{r_v^2}\right) dr$$

$$= \int_0^\infty \frac{r}{\sigma^2} \exp\left[- r^2 * \left(\frac{1}{2\sigma^2} + \frac{1}{r_v^2}\right)\right] dr$$

Using integration by parts, let $-r^2\left(\dfrac{1}{2\sigma^2} + \dfrac{1}{r_v^2}\right) = u$.

Differentiation yields $-2r\left(\dfrac{1}{2\sigma^2} + \dfrac{1}{r_v^2}\right)dr = du$

therefore

$$P_{KSS} = \frac{1}{2\sigma^2\left(\dfrac{1}{2\sigma^2} + \dfrac{1}{r_v^2}\right)} \int_0^{r_v} \exp(u)\ du =$$

or

$$P_{KSS} = \frac{1}{2\sigma^2\left(\dfrac{1}{2\sigma^2} + \dfrac{1}{r_v^2}\right)}\ (-\exp u)\Big|_0^{r_v}$$

$$= \frac{1}{2\sigma^2\left(\dfrac{1}{2\sigma^2} + \dfrac{1}{r_v^2}\right)} * \left[-\exp\left[-r^2\left(\dfrac{1}{2\sigma^2} + \dfrac{1}{r_v^2}\right)\right]\right]_0^{r_v}$$

$$= \frac{1}{1 + \dfrac{2\sigma^2}{r_v^2}} * \left[-0 + 1\right] = \frac{r_v^2}{r_v^2 + 2\sigma^2} \quad \text{and for}$$

$$A_v = \pi r_v^2$$

$$= \frac{A_v/2\pi}{\dfrac{A_v}{2\pi} + 2\sigma^2}$$

(3) Evaluation of equation (14).

$$P_{KSS} = \int\int_{-\infty}^{\infty} P_H(x,y) * P_{K/H}(x,y)\ dxdy$$

$$P_H = \frac{1}{2\pi\sigma_{x_2}\sigma_{y_2}}\ exp\left\{-\frac{1}{2}\left[\frac{(x-\mu_x)^2}{\sigma_{x_2}^2} + \frac{(y-\mu_y)^2}{\sigma_{y_2}^2}\right]\right\}$$

$$P_{K/H} = exp\left\{-\frac{1}{2}\left[\left(\frac{x}{\sigma_{x_1}}\right)^2 = \left(\frac{y}{\sigma_{y_1}}\right)^2\right]\right\}$$

Thus

$$P_{KSS} = \int\int_{-\infty}^{\infty} \frac{1}{2\pi\sigma_{x_2}\sigma_{y_2}}\ exp\left\{-\frac{1}{2}\left[\left(\frac{x-\mu_x}{\sigma_{x_2}}\right)^2 + \left(\frac{y-\mu_y}{\sigma_{y_2}}\right)^2\right]\right\}$$

$$exp\left\{-\frac{1}{2}\left[\left(\frac{x}{\sigma_{x_1}}\right)^2 + \left(\frac{y}{\sigma_{y_1}}\right)^2\right]\right\}$$

Expanding and combining terms yields,

$$P_{KSS} = \frac{exp\left\{-\frac{1}{2}\left[\left(\frac{\mu_x}{\sigma_{x_2}}\right)^2 = \left(\frac{\mu_y}{\sigma_{y_2}}\right)^2\right]\right\}}{2\pi\sigma_{x_2}\sigma_{y_2}}$$

$$* \int\int_{-\infty}^{\infty} exp\left\{-\frac{1}{2}\left[\left(\frac{1}{\sigma_{x_1}^2} + \frac{1}{\sigma_{x_2}^2}\right)x^2 - \left(\frac{2\mu_x}{\sigma_{x_2}^2}\right)x\right.\right.$$

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$$+ \left(\frac{1}{\sigma_{y_1}^2} + \frac{1}{\sigma_{y_2}^2} \right) y^2 - \left(\frac{2\mu_y}{\sigma_{y_2}^2} \right) y \right] dxdy$$

Separating variables and making substitutions yields,

$$P_{KSS} = \frac{\exp\left\{ -\frac{1}{2}\left[\left(\frac{\mu_x}{\sigma_{x_2}} \right)^2 + \left(\frac{\mu_y}{\sigma_{y_2}} \right)^2 \right] \right\}}{2\pi\sigma_{x_2}\sigma_{y_2}}$$

$$* \int_{-\infty}^{\infty} \exp\left[- p_x^2 * x^2 + q_x * x \right] dx \int_{-\infty}^{\infty} \exp\left[- py^2 * y^2 + qy * y \right] dy \quad (2)$$

where

$$p_x^2 = \frac{1}{2} \left(\frac{1}{\sigma_{x_1}^2} + \frac{1}{\sigma_{x_2}^2} \right)$$

$$p_y^2 = \frac{1}{2} \left(\frac{1}{\sigma_{y_1}^2} + \frac{1}{\sigma_{y_1}^2} \right)$$

$$q_x = \frac{\mu_x}{\sigma_{y_2}}$$

$$q_y = \frac{\mu_y}{\sigma_{y_2}}$$

Integrating each integrand I_x and I_y yields

$$I_x = \int_{-\infty}^{\infty} \exp\left\{-p_x^2*x^2 + q_x*x\right\} dx = \int_{-\infty}^{\infty} \exp\left\{-\left(p_x^2*x^2 - q_x*x\right.\right.$$

$$+ \frac{q_x^2}{4p_x^2} - \frac{q_x^2}{4p_x^2}\right\} dx$$

$$= \int_{-\infty}^{\infty} \exp\left\{-\left(p_x*x - \frac{q_x}{2p_x}\right)^2 + \frac{q_x^2}{4p_x^2})^2\right\} dx$$

$$= \exp\left(\frac{q_x^2}{4p_x^2}\right) \int_{-\infty}^{\infty} \exp\left\{-\left(p_x*x - \frac{q_x}{2p_x}\right)^2\right\} dx$$

Let $u = p_x*x - \dfrac{q_x}{2p_x}$ then $du = p_x dx$

then $I_x = \exp\left(\dfrac{q_x^2}{4p_x^2}\right) * \dfrac{1}{p_x} \displaystyle\int_{-\infty}^{\infty} \exp(-u^2)\ du$

But in general, $\dfrac{1}{\sqrt{2\pi}} \displaystyle\int_{-\infty}^{\infty} \exp\left\{-\frac{1}{2} z^2\right\} dz = 1$ or

$$\int_{-\infty}^{\infty} \exp\left\{-\frac{1}{2} z^2\right\} dz = \sqrt{2\pi}$$

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Let $u^2 = \frac{1}{2} z^2$ then $du = \frac{1}{\sqrt{2}} dz$ and

$$\sqrt{2} \int_{-\infty}^{\infty} \exp\left\{ - u^2 \right\} du = \sqrt{2\pi} \quad \text{or}$$

$$\int_{-\infty}^{\infty} \exp\left\{ - u^2 \right\} du = \sqrt{\pi}$$

therefore $I_x = \exp \dfrac{q_x^2}{4p_x^2} * \dfrac{1}{p_x} * \sqrt{\pi}$ or

$$I_x = \frac{\sqrt{\pi}}{\sqrt{\frac{1}{2}\left(\frac{1}{\sigma_{x_1}^2} + \frac{1}{\sigma_{x_2}^2}\right)}} \exp\left\{ \frac{\dfrac{\mu_x^2}{\sigma_{x_2}^2}}{4\left[\frac{1}{2}\left(\frac{1}{\sigma_{x_1}^2} + \frac{1}{\sigma_{x_2}^2}\right)\right]} \right\}$$

$$I_x = \frac{\sqrt{\pi}}{\sqrt{\frac{1}{2}\left(\frac{1}{\sigma_{x_1}^2} + \frac{1}{\sigma_{x_2}^2}\right)}} \exp\left\{ \frac{\mu_x^2}{2\sigma_{x_2}^2\left(\frac{\sigma_{x_2}^2 + \sigma_{x_1}^2}{\sigma_{x_1}^2 \sigma_{x_2}^2}\right)} \right\} \quad \text{or}$$

$$I_x = \frac{\sqrt{2\pi}\,\sigma_{x_1}\sigma_{x_2}}{\sqrt{\sigma_{x_1}^2 + \sigma_{x_x}^2}} \exp\left\{ \frac{\mu_x^2 \sigma_{x_1}^2}{2\sigma_{x_2}^2\left(\sigma_{x_1}^2 + \sigma_{x_2}^2\right)} \right\}$$

similarly

$$I_y = \frac{\sqrt{2\pi}\ \sigma_{y_1}\sigma_{y_2}}{\sqrt{\sigma_{y_1}^2 + \sigma_{y_2}^2}}\ \exp\left\{\frac{\mu_y^2\sigma_{y_1}^2}{2\sigma_{y_2}^2\left(\sigma_{y_1}^2 + \sigma_{y_2}^2\right)}\right\}$$

Substituting I_x and I_y into equation (2) yields

$$P_{KSS} = \frac{\exp\left\{-\frac{1}{2}\left[\left(\frac{\mu_x}{\sigma_{x_2}}\right)^2 + \left(\frac{\mu_y}{\sigma_{y_2}}\right)^2\right]\right\}}{2\pi\sigma_{x_2}\sigma_{y_2}} * \frac{\sqrt{2\pi}\ \sigma_{x_1}\sigma_{x_2}}{\sqrt{\sigma_{x_1}^2 + \sigma_{x_2}^2}}$$

$$* \frac{\sqrt{2\pi}\ \sigma_{y_1}\sigma_{y_2}}{\sqrt{\sigma_{y_1}^2 + \sigma_{y_2}^2}} * \exp\left\{\frac{\mu_x^2\sigma_{x_1}^2}{2\sigma_{x_2}\left(\sigma_{x_1}^2 + \sigma_{x_2}^2\right)}\right\}$$

$$* \exp\left\{\frac{\mu_y^2\sigma_{y_1}^2}{2\sigma_{y_2}\left(\sigma_{y_1}^2 + \sigma_{y_2}^2\right)}\right\}$$

or $\quad P_{KSS} = \dfrac{\sigma_{x_1} * \sigma_{y_1}}{\left(\sqrt{\sigma_{x_1}^2 + \sigma_{x_2}^2}\right) * \left(\sqrt{\sigma_{y_1}^2 + \sigma_{y_2}^2}\right)}$

$$\exp\left\{-\frac{1}{2}\left[\frac{\mu_x^2}{\sigma_{x_1}^2} - \frac{\mu_x^2\sigma_{x_1}^2}{\left(\sigma_{x_1}^2 + \sigma_{x_2}^2\right)\sigma_{x_2}}\right]\right\}$$

$$* \exp \left\{ - \frac{1}{2} \left[\frac{\mu_y^2}{\sigma_{y_2}^2} - \frac{\mu_y^2 \sigma_{y_1}^2}{\left(\sigma_{y_1}^2 + \sigma_{y_2}^2 \right) \sigma_{y_2}^2} \right] \right\}$$

or

$$P_{KSS} = \frac{\sigma_{x_1} * \sigma_{y_1}}{\left(\sqrt{\sigma_{x_1}^2 + \sigma_{x_2}^2} \right) \left(\sqrt{\sigma_{y_1}^2 + \sigma_{y_2}^2} \right)}$$

$$* \exp \left\{ - \frac{1}{2} \left[\frac{\mu_x^2}{\sigma_{x_1}^2 + \sigma_{x_2}^2} + \frac{\mu_y^2}{\sigma_{y_1}^2 + \sigma_{y_2}^2} \right] \right\}$$

If we assume that $\sigma_{x_1} = \sigma_{y_1}$ and $A_v = 2\pi \sigma_{x_1} \sigma_{y_1}$

the P_{KSS} becomes

$$P_{KSS} = \frac{A_v / 2\pi}{\sqrt{\sigma_{x_2}^2 + \frac{A_v}{2\pi}} \sqrt{\sigma_{y_2}^2 + \frac{A_v}{2\pi}}}$$

$$* \exp \left\{ - \frac{1}{2} \left[\frac{\mu_x^2}{\sigma_{x_2}^2 + \frac{A_v}{2\pi}} + \frac{\mu_y^2}{\sigma_{y_2}^2 + \frac{A_v}{2\pi}} \right] \right\}$$

or

$$P_{KSS} = \frac{A_v / 2\pi}{\sqrt{\sigma_x^2 + \frac{A_v}{2\pi}} \sqrt{\sigma_y^2 + \frac{A_v}{2\pi}}}$$

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$$* \exp\left\{-\frac{1}{2}\left[\frac{\mu_x^2}{\sigma_x^2 + \frac{A_v}{2\pi}} + \frac{\mu_y^2}{\sigma_y^2 + \frac{A_v}{2\pi}}\right]\right\}$$

$\overline{P}_K$ CALCULATIONS THROUGH THE TRANSITION MATRIX METHOD

TABLE 2

Computation of the Transition Matrix (T) for Example A/C #3

Probability of Transitioning from this State   to this State

	Knrc	Krcl	Krc2	Krc	Nk	
	1	$\dfrac{5+24+15}{530}$	$\dfrac{5+24+15}{530}$	0	$\dfrac{5+24+15}{530}$	Knrc
	0	$\dfrac{24+438}{530}$	0	0	24/530	Krcl
$T_3 =$	0	0	$\dfrac{24+438}{530}$	0	24/530	Krc2
	0	24/530	24/530	1	0	Krc
	0	0	0	0	438/530	Nk

(Note that the sum of each column is unity.)

Now, let the probability that the aircraft exists in each of
of the five possible states after the jth hit be expressed
by a vector $\{S\}^{(j)}$, where

$$\{S^{(j)}\} = \begin{Bmatrix} Knrc \\ Krcl \\ Krc2 \\ Krc \\ Nk \end{Bmatrix}^{(j)}$$

The probability that the aircraft is in each of the five states after the $(j+1)$th hit is

$$\{S\}^{(j+1)} = [T]\{S\}^{(j)} \qquad (B-1)$$

that is, the aircraft transitions from $\{S\}^{(j)}$ to $\{S\}^{(j+1)}$ to $[T]$. Thus, the cumulative probability the aircraft is killed after $j+1$ hit, $\bar{P}_K^{(j+1)}$, is given by

$$\bar{P}_K^{(j+1)} = Knrc^{(j+1)} + Krc^{(j+1)}$$

Prior to the first hit, the aircraft is entirely in the Nk state. Thus, according to equation (B-1),

$$\{S\}^{(1)} = [T]\{S\}^{O} = [T] \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{Bmatrix}^{(0)} \leftarrow Nk$$

and hence,

$$\{S\}^{(1)} = \begin{Bmatrix} 44/530 \\ 24/530 \\ 24/530 \\ 0 \\ 438/530 \end{Bmatrix}^{(1)} = \begin{Bmatrix} .0830 \\ .0452 \\ .0452 \\ 0 \\ .8264 \end{Bmatrix}^{(1)} \begin{matrix} \leftarrow Knrc \\ \\ \\ \leftarrow Krc \\ \end{matrix}$$

Thus $\bar{P}_{K_1}^{(1)} = .0830.$

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Similarly, for the second hit,

$$\{s\}^2 = [T]\{s\}^1 = [T] \begin{Bmatrix} .0830 \\ .0452 \\ .0452 \\ 0 \\ .8264 \end{Bmatrix} \qquad (1)$$

$$\{s\}^{(2)} = \frac{1}{530^2} \begin{Bmatrix} 530*44+44*24+44*24+44*438 \\ 462*24+24*438 \\ 462*24+24*438 \\ 24*24+24*24 \\ 438*438 \end{Bmatrix}$$

$$\{s\}^{(2)} = \begin{Bmatrix} .1591 \\ .0769 \\ .0769 \\ .0041 \\ .6829 \end{Bmatrix}^{(2)} \begin{matrix} \longleftarrow Knrc \\ \\ \\ \longleftarrow Krc \\ \end{matrix}$$

Thus, after the second hit

$$\bar{P}_K^{(2)} = .1591 + .0041$$

Similarly, the values for $\bar{P}_K^{(3)}$, $\bar{P}_K^{(4)}$, etc., are obtained to yield Table 1.

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APPENDIX C

P001 'FIRE' SECTION

```
703  C
704  C       COMPUTE SURVIVAL AND KILL PROBABILITIES
705  C
706     STUFF=DIVIDE(BXF2,(SXF2+AVTPI))+DIVIDE
        (BYF2,(SYF2+AVTPI))
707  C
708  CC PKS(IJ) IS THE PROBABILITY OF KILLING COMPONENT IJ
        BY A SINGLE ROUND
709  CC (ONE BARREL)
710  C
711         PKS(IJ)=AMIN1(1.0,DIVIDE(EXP(-.5*STUFF)*AVTPI,
712         SQRT((SXF2+1AVTPI)*(SYF2+AVTPI))))
713  C
714  CC PK IS THE PROBABILITY OF KILLING AT LEAST ONE
        COMPONENT BY A SINGLE
715  CC ROUND (ONE BARREL)
716  C
717         PK=PK+PKS(IJ)-PK*PKS(IJ)
718  C
719  CC CPKS(IJ) IS THE CUMULATIVE PROBABILITY OF KILLING
        COMPONENT IJ
```

```
720  CC BY THIS GUN LOCATION FOR THE ENTIRE AIRCRAFT FLIGHT
        PATH
721  C
722      460 CPKS(IJ)=CPKS(IJ)+(1-(1-PKS(IJ))**ISB)-(CPKS(IJ)*
723      *(1-(1-PKS(IJ)**ISB))
724  C
725  CC PS IS THE PROBABILITY OF ALL COMPONENTS SURVIVING
        ISB BARRELS FIRING
726  C
727          PS=(1.0-PK)**ISB
728  C
729  CC PK IS THE PROBABILITY OF KILLING AT LEAST ONE
        COMPONENT BY ISB
730  CC BARRELS FIRING
731  C
732          PK=1.0-PS
733          TI=TIME+T
734  C
```

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